A general illuminance model for daylight availability

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Noting the below observations, it appears possible to avail ourselves of much of the useful research done in the past to aid us in developing a generic model for daylight availability across North America. Much can be learned of a general nature by reviewing the sky illuminance measurement of others, but to make this wealth of information useful for our particular interest in North American skies, which is our objective here, extensive sky measurement must be initiated for several strategic sites in the United States and Canada. Those made in this country need to be updated and new measurements need to be made.

Introduction

The amount of exterior daylight has the greatest uncertainty of any value used in daylighting calculations. Without knowledge of the available daylight to be expected at any given instant in time, it is nearly impossible to adequately design a daylight system. For our practice in North America, Kimball and Hand4 (1922) have provided an excellent reference source of sky measurements which has been used exclusively as the basis of the existing IES Daylighting Practice.2,3 Despite the well accepted use, however, of the existing daylight availability curves currently recommended by the IES of North America, these data have not been compared or evaluated against any of the extensive measurements made over the last 60 years, either in this country or other countries. After comparing the Kimball and Hand data with recent sky measurements made in this country, and with measurements made around the world since 1922, an improved general illuminance model for daylight availability is recommended which is applicable for most locations across North America.

Historical review

While the sky measurements made by Kimball and his associates (1919–1922) for Washington and Chicago are probably the best known in North America, similar measurements have been made by others within the United States. Sky brightness (not illuminance) was recorded as early as 1897 for Chicago by Basquin, A. and Kunerth and Miller3 (1932) studied visible and ultraviolet radiation on a horizontal surface for Ames, Iowa. Other illuminance measurements have been reported by Johnston6 (1939) for Pittsburgh, Boyd7 (1953–1954) for Ann Arbor, Michigan, and Kingsbury, Anderson, and Bizzaro8 (1953–1955) for Port Allegany, Pennsylvania. Also, the U.S. Weather Bureau9 (now a part of the National Oceanic and Atmospheric Administration) reported horizontal illumination in terms of a time metric, footcandle-hours, for Washington, D.C., and Baltimore, Maryland (1954).

In Europe, a substantial history of sky illuminance measurements can be found. In the United Kingdom alone, data have been recorded for Tedington10,11 (1923–1939) and later for Kew and Bracknell12 (1964–1973). Krochmann13 collated measurements made at several locations throughout Europe including sites in the USSR, Finland, France, Germany, and Austria; while Elvegard and Sjostedt14 have reported substantial measurements made in Sweden as well. Outside of Europe, Nakamura and Ok15 have published their own measurements for Nagoya, Japan with reference made to similar measurements in India and Nepal. And lastly, a sophisticated sky instrumentation program has been underway for several years in Pretoria, South Africa.16 With such an historical daylight availability reference base at hand, several points can be made initially regarding horizontal illuminance.

1. For a given sky type (either clear or overcast), significant variations are not seen in horizontal...
illuminance from location to location when plotted against solar altitude. If such variations exist, they are obscured by the fluctuation seen in the measurements within each locality.

2. Since the sky measurements made by different authors are basically similar, it is not surprising to see the expressions used to represent their data are also similar. In some cases where equations appear to be substantially different, plotting the equations show how similar they actually are.

3. Sky types are almost always studied separately.

4. Solar altitude is by far the most common driving function used for prescribing exterior illuminance. Solar altitude seems to be a value easily accessible to most practicing engineers and although a characteristic scatter is obvious in the correlation, other more complex approaches do not improve the correlation significantly.

Noting the above observations, it appears possible to avail ourselves of much of the useful research done in the past to aid us in developing a generic model for daylight availability across North America.

Current measurement work

Much can be learned of a general nature by reviewing the sky illuminance measurements of others, but to make this wealth of information useful for our particular interest in North American skies, which is our objective here, extensive sky measurements must be initiated for several strategic sites in the United States and Canada. Those made in this country need to be updated and new measurements need to be made.

Sky measurements have been underway at the National Bureau of Standards, with joint support from the National Fenestration Council, where hourly measurements of sky luminance, horizontal and vertical illuminance, and solar radiation (irradiance) are recorded\(^\text{17}\) for Gaithersburg, Maryland. Furthermore, sky measurements are now also underway at the Lawrence Berkeley Laboratory (Berkeley, California), the Solar Energy Research Institute (Golden, Colorado),\(^\text{18}\) and the Florida Solar Energy Center (Cape Canaveral, Florida). Although only a limited amount of sky data from these latter three locations are currently available, a fair assessment can still be made when comparing the NBS measurements with these data and with the international work. Together, this data base is used to establish an improved model for daylight availability.

**Direct beam illuminance**

A logical beginning point in the discussion of
daylight availability concerns the available extraterrestrial solar illuminance. The mean extraterrestrial illuminance can be derived for air mass zero by integrating the ASTM standard spectral irradiance curve of the solar constant while correcting for the Commission Internationale de l'Eclairage (CIE) standard eye response. By using the trapezoidal rule, the method of numeric integration used by ASTM, the mean extraterrestrial solar illuminance $E_{\text{sc}}$ is found as follows:

$$E_{\text{sc}} = K_m \int_{380}^{760} V_\lambda E_{\text{st}} d\lambda$$

where

- $K_m = \text{the international standard maximum spectral luminous efficacy, 683 Lm/Watt}$
- $V_\lambda = \text{the CIE standard photopic spectral eye response}$
- $E_{\text{st}} = \text{the standard ASTM solar spectral irradiance averaged over the small wavelength band d}\lambda$

The resultant value,

$$E_{\text{sc}} = 127.5 \text{ klux}$$

can be thought of as the daylighting equivalent of the solar constant, $1353 \text{ W/m}^2$ as established by ASTM.

To obtain the extraterrestrial solar illuminance, the mean value must be adjusted to account for the actual earth-sun distance at any point in time. The slightly elliptical orbit of the earth causes a predictable variation in the extraterrestrial radiation that can be approximated by the equation:

$$E_x = E_{\text{sc}} \left[1 + 0.033 \cos \left(\frac{360^* J}{365}\right)\right]$$

where

- $J = \text{the Julian date, from } J = 1 \text{ to } J = 365$
- $E_x = \text{the extraterrestrial illuminance on day } J.$

Continuing our effort to derive an expression for the solar illuminance inside the atmosphere, the concept of the photopic atmospheric extinction coefficient can be introduced. As with any radiation passing through an attenuating medium, the amount of direct illuminance that passes through the atmosphere can be represented by,

$$E_{\text{DN}} = E_x e^{-am}$$

where

- $E_{\text{DN}} = \text{the direct normal solar illuminance.}$
a = the optical atmospheric extinction coefficient.

m = the optical air mass.

While it is recognized that there are several ways of representing the air mass, the simplest (and by far the most common) form is

\[ m = \frac{1}{\sin h} \]  

where \( h \) is the solar altitude. Furthermore, if a single average value is used for the extinction coefficient, a manageable expression for the direct solar illuminance becomes,

\[ E_{DN} = E_{sc} \left[ 1 + 0.033 \cos \left( \frac{360^\circ \cdot t}{365} \right) \right] e^{-a/\sin h}. \]  

Making use of the data base previously established, along with measurements made of the direct normal illuminance at NBS (Figure 1) an average extinction coefficient \( a = 0.210 \) was obtained for clear sky conditions. Plots are shown in Figures 2, 3, and 4 of some of the data available by date showing the correlation between these measurements and Equation 4. Also plotted in Figures 3 and 4 are the limited number of values currently in the Reference Volume of the IES Handbook. It is as once obvious that the proposed expression is a substantial improvement over the existing values.

Jones and Condit performed a similar comparison. They extrapolated the Kimball and Hand data for horizontal solar illuminance with the sun at the zenith, and derived a value of 104.9 klux for the average condition between the December and June sun. They then compared this to Moon's proposed standard, which is 108.1 klux when revised to agree with the currently recommended value of \( K_m \). From Equation 4 the predicted equinox illuminance is 105.3 klux, which agrees with the Jones and Condit data by under 1 percent and with Moon by within 3 percent. Similarly, Elvegard, and Sjöstedt fit a curve through the Swedish and Finnish data and reported a constant extinction coefficient of 0.231; a value that would cause the direct solar illuminance to be only slightly lower than predicted by Equation 4.

**Diffuse sky illuminance**

As sunlight passes through the atmosphere, a portion of the incident radiation is scattered by dust, water vapor, and other suspended particles in the atmosphere. This scattered, or diffuse, light provides a substantial amount of daylight and is normally divided into three categories: overcast, clear, and
partly cloudy. For our purposes we will distinguish the three sky types according to the sky ratio, the ratio of the diffuse to total horizontal illuminance. Therefore, a sky ratio of 0.80 or greater is assumed overcast; a ratio of less than 0.28 clear; and ratios in between partly cloudy.

The work of various researchers has resulted in a series of proposed equations for each of the three types of diffuse sky (Table 1). While it is recognized that such factors as atmospheric turbidity, local atmospheric pressure, cloud type, cloud amount, and snow cover all affect the sky illuminance, in every case the solar altitude has been singled out as the primary driving function. The other parameters appear to have only a second order effect, and therefore, can usually be ignored.

Figure 5 plots the diffuse clear sky illuminance as a function of solar altitude, and shows not only the consistency of the measurements, but also a consistency for other locations. Two facts appear by comparing the clear sky data in Figure 5 with Equation 5, generic values for A, B, and C give,

$$E_d = A + B \sin h C$$

where A, B, and C are empirically defined constants. Nagel\(^2\) and Angström and Drummond\(^3\) promote the incorporation of the turbidity coefficient $\beta$, but since it is normally an unknown, and because there is substantial uncertainty in its use, $\beta$ was not included as a part of Equation 5. In comparing the clear sky data in Figure 5 with Equation 5, generic values for A, B, and C give,

$$E_{dclr} = 0.8 + 15.5 (\sin h)^{0.5} \text{ (in klux)}$$

which is virtually the Krochmann equation\(^2\) with a slight adjustment for the sunrise/sunset illuminance.

Looking at how this expression compares to the existing values given in the IES Reference Volume (excluding the curve fit which was not a part of the original data), it is clear that these data are still representative of the larger data base. Nevertheless, if an equation is desired to represent the data, equation 6 shows itself more representative of the larger data base and should be applicable for other locations as well. Analyzing the overcast condition is more difficult. The presence of clouds, even when
they cover the complete sky, causes the sky to be less stable. Figure 6 shows this. The scatter as seen is characteristic for all locations and is due to differing cloud densities, cloud heights, and water vapors, all of which are continuously changing. Snow cover also has a significant effect. Kaftin showed that cloud type and snow cover alone could vary the overcast horizontal illuminance by as much as 300 percent. Yet it may still be possible to draw some general conclusions. Referring again to Table 1b we observe that several equations have been proposed for the illuminance of the overcast sky. However, it is evident that most of these expressions plot the diffuse illuminance in approximately the same way, and given the scatter in the data, imply the sufficiency of the more simple forms. The solid curve in Figure 6 is the Krochmann overcast equation, which while being probably the best fit to the measurements, is also among the simplest. Furthermore, the values used in the IES Reference Volume are shown, and demonstrate that this equation is in general agreement with the IES reference data as well. Therefore, a recommended equation for the overcast sky is,

$$E_{disc} = 0.30 + 21.0 \sin h \text{(in klux)}$$  \hspace{1cm} (7)

**Partly cloudy sky illuminance**

The partly cloudy sky is perhaps the most difficult to analyze since it is considered the least stable. However, by first looking only at the diffuse component, much of the instability is overcome and a fairly consistent correlation with solar altitude becomes apparent. The presence of a few clouds in an otherwise clear blue sky increases the diffuse sky intensity substantially: both Jones and Condit, and Kimball and Hand have reported partly cloudy skies as being three to four times the diffuse clear sky intensity. Figure 7 confirms this. As haze, or the amount of cloud cover increases, the diffuse illuminance initially jumps to values reaching or exceeding the overcast sky illuminances until the cloud cover completely obstructs the sun and the overcast sky condition is reached.

The current IES Recommended Practice does not cover partly cloudy conditions, and only a few equations for such a sky type have been proposed by others (Table 1c), most of which fall within the band of the measurements shown in Figure 7. Therefore, because most of the suggested equations plot the idea in a similar way, and because the general form of Equation 5 is still apparent among these equations, an expression analogous with the clear and overcast sky equations can be written. For the partly cloudy sky Equation 5 becomes,

$$E_{dpc} = 0.3 + 45.0 \sin h \text{(in klux)}.$$  \hspace{1cm} (8)
cate a direct beam illuminance of 3 percent of its unobstructed value under a light haze, 0.1 percent under a medium haze, and 0.003 percent under a moderately heavy haze. Elvegard and Sjöstedt have suggested values more in the range of 35 percent when the direct sun is obscured by a thin cloud film. Both extremes appear to be valid. From the definition of the partly cloudy sky, the solar disc can range from being completely exposed to almost completely obstructed. It is therefore necessary to allow a substantial range of direct beam illuminances.

Yet it may still be possible to provide a useful expression for the direct component in the partly cloudy sky. For purposes of a general illuminance model, it will be assumed that the atmosphere is homogeneous as seen by the solar disc, allowing the direct normal illuminance to be computed from Equation 4. The atmospheric extinction coefficient can be determined indirectly from the NBS measurements as follows. For each partly cloudy measurement of diffuse illuminance $E_d$ and total illuminance $E_t$, the expressions

$$B = \frac{E_d - 0.3}{\sin h}$$

and

$$a = -\ln \left( \frac{E_t - E_d}{127.5 \sin h} \right) \sinh$$

(10)

can be used to find values for the partly cloudy sky coefficient $B$ and the atmospheric extinction coefficient $a$. Equation 9 is obtained from Equation 5 with values for $A$ and $C$ from Equation 8. Equation 10 is obtained by solving Equation 4 for the extinction coefficient, using a mean value for the extraterrestrial illuminance.

Plots from the NBS measurements of the coefficients $a$ and $B$ are shown in Figure 8. The variation in the data shows the characteristic dynamics of the partly cloudy sky. However, the data can be bounded. Representing the bounds in terms of the 94 percent confidence level (94 percent of all measurements fall between the two curves), values for the upper and lower bounds become,

$$a_{\text{max}} = -\ln \left( \frac{79.5 - B}{127.5} \right)$$

(11)

and

$$a_{\text{min}} = -\ln \left( \frac{129.0 - B}{127.5} \right)$$

(12)
OVERCAST SKY DIFFUSE ILLUMINANCE

Figure 6. Overcast sky diffuse illuminance as a function of solar altitude for several locations.

Table 2. General illuminance model equation parameters

<table>
<thead>
<tr>
<th>Sky</th>
<th>a</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clear</td>
<td>0.21</td>
<td>0.8</td>
<td>15.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Partly cloudy</td>
<td>0.80</td>
<td>0.3</td>
<td>45.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Overcast</td>
<td>*</td>
<td>0.3</td>
<td>21.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

*Overcast condition assumes direct sun is completely obstructed

Thus, for B = 45, as used in Equation 8, the atmospheric extinction coefficient has a range from 0.4 to 1.3. The median value for this range is 0.80, which would probably be the best single value for the partly cloudy condition.

Table 2 summarizes suggested values of a, A, B, and C as used in Equations 4 and 5.

Total horizontal illuminance

It is now possible to combine the direct and diffuse components into a single expression for the total horizontal illuminance:

For the clear sky—

$$E_h = E_{D_{Na=0.21}} \sin h + E_{dfr}$$  \hspace{1cm} (13)

For the overcast sky—

$$E_h = E_{D_{Na}}$$  \hspace{1cm} (14)

For the partly cloudy sky—

$$E_h = E_{D_{Na=0.80}} \sin h + E_{dpc}$$  \hspace{1cm} (15)

Conclusions

A generic illuminance model, in the form of a few equations, has been developed for determining the available horizontal daylight from the direct sun and diffuse sky. Being based on measurements recently made within the United States and elsewhere, the equations can be used for other locations in North America with a fair amount of confidence. It is important to remember, however, that the equations are availability models representing mean conditions for design calculations, and that instantaneous values could vary substantially. This can be seen from Figures 5–7. Also, the equations are a function only of solar altitude and do not account for many of the other atmospheric factors. But this was deliberate. Solar altitude is normally the only quantity accessible to lighting designers: tables can be found in the Reference Volume of the IES Handbook.
Figure 7. Partly cloudy diffuse illuminance as a function of solar altitude for locations within the United States.

Figure 8. Partly cloudy extinction coefficient as a function of the sky coefficient B.
tions useful in obtaining solar altitude, sources such as Duffie and Beckmann can be consulted.

In comparing the existing IES values with the larger and more recent data base, the values for diffuse sky illuminance were shown to be similar, but the direct sun intensities appear low. Therefore, the developed equations were found to be more representative of the direct measurements and consistent with the existing diffuse measurements. Since equations are becoming necessary as calculators and computers are becoming commonplace, the models developed here are more preferable and complete than the IES values in predicting the availability of exterior daylight.

References


Rebuttal

The authors appreciate Dr. Murdoch’s comments. They will be referred to by number.

1. As Dr. Murdoch suggests, the cosine term used to correct the extraterrrestrial solar illuminance is not a critical one. However, it is a very predictable variation and one that is normally included in solar radiation (irradiance) calculations, and for this reason it was included in Equations 1 and 4.

2. The angle correction mentioned for determining direct solar illuminance on a horizontal plane is found in Equations 13 and 14 where the total horizontal illuminance is computed. Since vertical illuminance was not a part of this paper (although it could have been) the vertical angle correction was not mentioned.

3. Dr. Murdoch is correct, the procedure for categorizing sky types requires more discussion. Let us begin by pointing out a common misconception regarding sky conditions, particularly overcast conditions. When the overcast sky is defined as one with 100 percent cloud cover, does this refer to a sky where the blue sky is completely obstructed by some type of cloud cover? If so, does not fog or dense haze also fit this definition? And if the stipulation is further made that the sun is
not visible, does this mean that the sun is not directly visible or that it is completely imperceptible? If the former, then indirect sky shine must be included; if the latter, than storm and rain conditions must be included. If the CIE definition is followed prescribing the symmetric standard luminance condition, virtually no measurements can be included. This obviously causes a dilemma. Few sky conditions are perfectly diffuse and uniformly overcast, yet many are close. By experience—by looking at a lot of sky photographs and viewing a lot of sky conditions, we have noted that once the sky ratio exceeds 0.90 the sun is virtually imperceptible or at least closely so. Also, while it is possible to weed out obvious storm conditions, light drizzle is impossible to detect by computer and probably accounts for some of the low values plotted in Figure 6. In summary, distinguishing sky types is not as straightforward as one might guess. Even with the use of values like sky ratio and fraction of cloudiness, experience and human judgement are still necessary.

4. In suggesting the use of the standard luminance distribution equations for determining horizontal illuminance, it should be pointed out that such an approach suffers from the same weaknesses as the one taken by the authors. Namely, that an empirical equation based on solar altitude is necessary. That equation can be either to obtain the zenith luminance or integrated illuminance. In fact, we might mention an earlier study we conducted on modeling zenith luminance, and for the overcast sky mentioned by Dr. Murdoch, we actually found a poorer correlation, especially when plotting Krockmann’s equation—which should not be interpreted as a criticism of his equation, but rather of such a plot. Using integrated values does help the correlation somewhat.

5. Again, regarding the clear sky luminance, it is still necessary to obtain zenith luminance first. But here the story does change. We have found the zenith luminance much more stable, and thus, much better a driving function. As suggested, this alternate approach could be taken; yet as is also suggested, it appears to show about the same results.

References