LLUMINATING engineers are becoming increasingly conscious of the necessity for planned lighting that will provide ideal visual conditions. As a step in this direction the I.E.S. Committee on Standards of Quantity and Quality of Interior Illumination has recommended a 3:1 brightness (helios) ratio for all rooms in which the visual task is severe. The recently developed interreflection method allows the brightness (helios) ratios to be predetermined for any room and gives a foundation for the design of really satisfactory lighting installations. A previous paper presented a series of charts showing under what conditions the 3:1 ratio can be satisfied. Exposed fluorescent lamps give a brightness (helios) ratio of 50:1 or more and therefore should not be used in offices, drafting rooms, school rooms, or other places where close visual work is encountered. The same brightness (helios) ratio is obtained with louvered fixtures and with the overglass ceiling, which are therefore potential sources of reflected glare.

Ideal lighting is thus narrowed down to two possibilities:

1. Ceiling lighting, Type IIb, where the entire ceiling acts as a secondary source. The only satisfactory hanging luminaire for this type of lighting is of the luminous indirect type, the bottom of the luminaire having approximately the same brightness (helios) as the ceiling.

2. Luminous-ceiling lighting, Type IIa, utilizing a hung ceiling of translucent plastic or other diffusing material lighted from above.

Even with IIa and IIb lighting, the 3:1 criterion is satisfied only if high enough reflectances are used. With a floor reflectance of 0.10, the interreflection method (Fig. 1a) shows that the 3:1 ratio is never satisfied, though a ratio of less than 10:1 is usually obtained. By raising the floor reflectance to 0.30 and employing a wall reflectance of at least 0.50, however, one obtains excellent visual conditions (Fig. 1b), at the same time utilizing the light most economically.

Type IIb lighting is employed in many installations today, though generally with such low reflectances that the potential excellence of the lighting system is obscured. Luminous-ceiling lighting (IIa) offers interesting possibilities and merits further study. The purpose of the present paper is to begin such a study by investigating the design of luminous ceilings.

Fig. 2 shows one arrangement for ceiling lighting. Continuous rows of fluorescent lamps are attached to the ceiling. No reflectors are employed and the ceiling is painted white. At distance l below the lamps is a grid of metal bars. Since the grid supports only a small weight and is hung from the ceiling at frequent intervals, it can be of very light construction. Resting on the grid are flat panels of diffusing plastic which can be slid to the side or removed for relamping and cleaning. A less-expensive construction employs, in place of each plastic panel, a simple frame covered with thin diffusing plastic, tracing paper, or glass cloth. Convenient dimensions are $s = 0.6m$ (24 in.), $l =$

*The 3:1 ratio does not mean that the maximum brightness (helios) in the room is limited to 3 times the minimum. It does mean that the adaptation brightness (helios), for any orientation of the eyes, must not exceed 3 times the adaptation brightness $H$ for the work; and the adaptation brightness of the work must not exceed 3 times the minimum adaptation brightness for any orientation of the eyes. Since the adaptation brightness associated with an extended surface is essentially equal to the brightness of the surface, one may express the criterion as

$$H_{\text{max}}/H_A < 3,$$

$$H_A/H_{\text{min}} < 3.$$

**The 40-watt T-12, 3500K white fluorescent lamp has a brightness (helios) of 19,000 blum and the 96-in., T-8, 4500K white lamp has approximately the same brightness (helios). But this value is so much greater than any other brightness (helios) in the room that poor visual conditions are sure to result if lowered fixtures are used. A high value of illumination (incident pharosage) is 500 lumens $m^{-2}$ (47 lumens ft$^{-2}$); and with ordinary printed matter having an average reflectance of 0.65, the eyes will adapt to the brightness (helios) $H_D = 500 \times 0.65 = 325$ blum. Thus the brightness (helios) ratio is $H_{\text{max}}/H_A \approx 18000/325 = 55$.**
0.3m (12 in.). The individual panels can be approximately 0.6 x 0.6m (24 x 24 in.). The appearance of a room lighted by this type of construction is indicated in Fig. 3.

**Design**

For any type of lighting, the average illumination (pharosage) is expressed by the well-known equation,

\[ D_{AV} = k_u \frac{F_L}{S} = f g \frac{F_L}{S} \]

where \( D_{AV} \) = average incident illumination (pharosage) (lumen \( m^{-2} \)) on principal surface,

\( k_u \) = coefficient of utilization,

\( F_L \) = total lumen (pharos) from lamps,

\( S \) = floor area \((m^2)\),

\( f \) = interfaction of room,

\( g \) = efficiency (logance) of luminaire.

For simplicity, we shall consider only rooms that are so large \((k_r \to 0)\) that the effect of the walls may be neglected. Further analysis\(^6\) shows that the general conclusions obtained in this limiting case apply to all rooms though numerical values may vary somewhat. For an infinite room lighted by a luminous ceiling,

\[ f = \frac{1}{1 - \frac{\rho_2}{\rho_3}} \]

Thus from Equation (1),

\[ D_{AV} = \frac{g F_L}{1 - \frac{\rho_2}{\rho_3}} \frac{1}{S} \]

Also, the brightness (helios) of ceiling and floor are

\[ H_2 = \frac{g F_L}{(1 - \frac{\rho_2}{\rho_3}) S} \]

\[ H_3 = \frac{\rho_2 g F_L}{(1 - \frac{\rho_2}{\rho_3}) S} \]

where \( \rho_2 \) = apparent reflectance of luminous ceiling, as viewed from below,

\( \rho_3 \) = reflectance of floor,

\( H_2 \) = average brightness (helios) of luminous ceiling, as viewed from below,

\( H_3 \) = average brightness (helios) of floor.

Equations (2) to (5) were obtained by mentally following the light as it bounces about between floor and ceiling.\(^7\) The interferences in the luminaire (upper portion of the room, including translucent plates, Fig 4) were not considered specifically. We now bring the luminaire into the picture and find how its characteristics change as the reflectance and transmittance are altered. The equation for \( D_{AV} \), derived in Appendix A of the present paper, is

\[ D_{AV} = \frac{\tau (1 + \rho_3)}{2 (1 - \rho_2 \rho_3)} \frac{1}{1 - \rho_3} \frac{1}{1 - \rho_2 \rho_3} \frac{F_L}{S} \]

(6)

Here \( \tau \) = transmittance of translucent plates,

\( \rho_2 \) = reflectance of translucent plates,

\( \rho_3 \) = reflectance of ceiling (See Fig. 4).

Comparison of equations (3) and (6) shows that the efficiency (logance) of the luminaire is
from equations (7) and (8). If \( k_c \to 0 \), equations (7) and (8) still give approximate values of efficiency (logance) and apparent reflectance which can be used in the interreflection tables.\(^8\)

Fig. 5 shows how the apparent reflectance of the translucent plates varies as the transmittance is varied. The assumption is made that

\[
\rho_2' = 0.90 - \tau,
\]

which agrees with experimental data on the best translucent materials.\(^9\) The curves were calculated by use of equation (8). If the ceiling were black (\( \rho_2 = 0 \)), the apparent reflectance \( \rho_2' \) would be the same as the reflectance \( \rho_2 \) of the translucent plate. When the ceiling is painted white, however, the apparent reflectance rises. For \( \tau = 0.50 \), the apparent reflectance is not equal to \( \rho_2 = 0.40 \) but varies from 0.65 to 0.80 depending on the reflectance \( \rho_2' \).

The efficiency (logance) of the luminaire is plotted in Fig. 6. The curves show again that the reflectance \( \rho_6 \) should be as high as possible. They also indicate that the transmittance of the translucent plates should be as high as is consistent with good diffusion.

Fig. 7 gives the coefficient of utilization for a room with ceiling lighting and having a floor reflectance of 0.30 and a \( k_c \) of zero. Note that a coefficient of utilization of 0.80 is easily obtainable,*

*Note that these values of \( k_c \) are for an infinite room. For other rooms, the coefficient of utilization is lower and can be obtained from the interreflection tables. Reference 9.
and values of more than 1.00 are possible by raising the reflectances $\rho_1$ and $\rho_2$. Thus luminous-ceiling lighting compares favorably with any other form of lighting in effectiveness. One might think that the louvered ceiling, being a "direct" lighting system, would have a higher coefficient of utilization than the luminous ceiling, but such is not the case. With a 45° cut-off, each cell of the louvered ceiling acts like a miniature cubical room interposed between the lamps and the large room. And lighting in a cubical room is always inefficient. This fact accounts for the low values of $k_a$ (0.25 to

Figure 7. Coefficient of utilization of an infinite room with HfA lighting, $k = 0$, $\rho_1 = 0.20$.

Figure 8. Brightness (helios) variation of the translucent plates for an infinite luminous ceiling, as affected by the lamp spacing $s$ and the distance $l$ between lamps and translucent plates. The contrast $c$ is

$$c = \frac{H_{\text{max}} - H_{\text{min}}}{H_{\text{max}}}.$$  

The horizontal line at $c = 0.10$ represents the minimum perceptible variation. Thus spacings of approximately $2l$ will give a ceiling that appears to be uniform.

0.40 according to C. L. Amick\(^{(10)}\) obtained experimentally on louvered installations.

Spacing of Lamps

Consider now the spacing between rows of lamps that will give visual uniformity for the luminous ceiling. Since the translucent plates are assumed to be perfectly diffusing, uniformity in brightness (helios) requires merely that the incident pharosage (including interfections) on the top of the plates be uniform. Appendix B gives equations for the maximum and minimum incident pharosage and for the contrast:

$$c = \frac{H_{\text{max}} - H_{\text{min}}}{H_{\text{max}}} = \frac{D_{\text{max}} - D_{\text{min}}}{D_{\text{max}}}.$$  

The results are plotted in Fig. 8, which shows how $c$ increases when the spacing between lamps is increased. The analysis includes the direct light from the lamps and the infinite number of interfections within the luminaire. According to W. E. K. Middleton,\(^{(11)}\) a contrast of 0.10 is just detectable in a gradual transition from one brightness to another. Using this criterion, we see that a spacing of $2l$ between rows of lamps is allowable. That is, in a very large room, the spacing between rows of fluorescent lamps may be made twice the distance from the lamps to the translucent plates. This rule presupposes a diffusing upper surface with high reflectance $\rho_2$.  

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Luminous Ceiling Lighting—Moon-Spencer  

ILLUMINATING ENGINEERING
TABLE I — Dimensions for Luminous-Ceiling Lighting
with normal spacing of \( s = 0.61 \text{m (24")}, \ i \geq 0.30 \text{m (12")}. \)

<table>
<thead>
<tr>
<th>Width of available space (( s )) (meter)</th>
<th>Number of rows (n)</th>
<th>Spacing of outside rows = ( \frac{1}{4} ) s (m)</th>
<th>Distance to sides of luminaire = ( \frac{1}{4} s ) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.91</td>
<td>3</td>
<td>2</td>
<td>0.15</td>
</tr>
<tr>
<td>1.52</td>
<td>5</td>
<td>3</td>
<td>0.15</td>
</tr>
<tr>
<td>1.83</td>
<td>6</td>
<td>4</td>
<td>0.15</td>
</tr>
<tr>
<td>2.44</td>
<td>8</td>
<td>5</td>
<td>0.15</td>
</tr>
<tr>
<td>2.86</td>
<td>10</td>
<td>6</td>
<td>0.15</td>
</tr>
<tr>
<td>3.66</td>
<td>12</td>
<td>7</td>
<td>0.15</td>
</tr>
<tr>
<td>4.27</td>
<td>14</td>
<td>8</td>
<td>0.15</td>
</tr>
<tr>
<td>4.68</td>
<td>16</td>
<td>9</td>
<td>0.15</td>
</tr>
<tr>
<td>5.49</td>
<td>18</td>
<td>10</td>
<td>0.15</td>
</tr>
<tr>
<td>6.10</td>
<td>20</td>
<td>11</td>
<td>0.15</td>
</tr>
</tbody>
</table>

For other spacings, change all dimensions in proportion. The total width is
\[ w = (n - \frac{1}{2}) s \] for \( n = 2 \) or 3,
\[ w = (n + 1) s \] for \( n \geq 4 \).

The distance from lamp to panel is
\[ l \geq \frac{s}{2} \].

The above study of lamp spacing has been for an infinite room. There still remains the question of lamp spacing at the sides of the room. Should the lamp spacing remain uniform throughout or should there be a reduction in spacing near the sides of the room? Also, what spacing should be employed between the final row of lamps and the side of the luminaire? The equations are given in Appendix C.

For a narrow panel containing only two rows of lamps, the illumination (pharosage) without interreflections is easily calculated. The results are shown in Fig. 9 for spacings of 1.5, 2, and 3 times the distance \( l \). The spacing \( s = 2l \) gives approximately 20 per cent variation without interreflections, but this value is reduced to about 10 per cent by multiple reflections. The graph shows also that to keep the edges of the translucent plate at sufficiently high helios, the distance between lamp and end of luminaire must be approximately one-fourth of the normal spacing between lamps. Similar conclusions apply to 3, 5, and 7 rows of lamps. A trial-and-error process showed that when more than three rows are employed, the outside rows should be moved toward the center so that their spacing is three-quarters of normal spacing. This arrangement is found to give better uniformity than with normal spacing \( (s = 2l) \) throughout. An illumination (pharosage) plot for seven lamps spaced in this way is shown in Fig. 10.

Conclusions

Equations and graphs have been given for the design of luminous ceilings. It is found that the coefficient of utilization compares favorably with the best values obtained with other lighting systems. In particular, the complete luminous ceiling costs no more than the louvered ceiling, has a higher coefficient of utilization, and eliminates that great defect of the louvered — reflected glare. Properly designed, the luminous ceiling gives ideal lighting that satisfies the 3:1 brightness (helios) criterion and all eight factors of lighting.12

Best results are obtained under the following conditions:

1. The translucent plates should have the highest transmittance that is consistent with complete diffusion. The optical absorbance of the material should be as low as possible.
2. The reflectance \( \rho \) of the upper surface in the luminaire (Fig. 4) should be at least 0.80.
3. Floor reflectance \( \rho \) in the room should be at least 0.30 and average wall reflectance should be at least 0.50.
4. The normal spacing \( s \) between rows of lamps should not exceed twice the distance \( l \) between lamp centers and translucent plates.
5. With more than 3 rows of lamps, the spacing of the outer rows should be three-quarters of normal spacing.
6. Spacing between outside rows of lamps and side walls of the luminaire should be one-fourth of normal spacing between lamps. The old rule has been to use one-half normal spacing, but the present analysis shows that one-fourth is definitely superior.

7. Each row of lamps should run to the extreme end of the luminaire. In fact better brightness uniformity is obtained by making the length of the luminous panel slightly less than the total length of a row of lamps.

Some suggested dimensions are given in Fig. 11 and Table I. The table shows how many rows $n$ of fluorescent lamps are needed for different available widths $w$. The tabulated values are for a normal spacing of 0.61 m (24 in.), but dimensions for any other spacing are easily obtained. The spacing $s$ is determined by the desired illumination (pharosage) $D_A$, and Table II is included as an aid in determining this spacing. Values of $D_A$ have been calculated for various lamp spacings and for two rooms, one very large ($k_r \to 0$) and one of rather small size ($k_r \approx 0.70$). Most rooms will fall between these two extremes and may be investigated in greater detail by use of the interreflection tables.

Table IIa gives the illumination (pharosage) in English units and Table IIb gives the same data in metric units.

---

**TABLE IIa — Average Illumination (Pharosage) as a Function of Lamp Spacing**

<table>
<thead>
<tr>
<th>Normal Spacing $*$ (in.)</th>
<th>Average Illumination (Pharosage) (lumen $m^{-1}$) in Service</th>
<th>$k_r \to 0$</th>
<th>$k_r = 0.70$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$T-12$, 40w</td>
<td>$T-8$, 200 ma</td>
<td>$T-8$, 100 ma</td>
</tr>
<tr>
<td>0.30 (12)</td>
<td>4060</td>
<td>2710</td>
<td>1600</td>
</tr>
<tr>
<td>0.40 (18)</td>
<td>2730</td>
<td>1800</td>
<td>1070</td>
</tr>
<tr>
<td>0.61 (24)</td>
<td>2640</td>
<td>1560</td>
<td>890</td>
</tr>
<tr>
<td>0.76 (30)</td>
<td>1640</td>
<td>1090</td>
<td>640</td>
</tr>
<tr>
<td>0.92 (36)</td>
<td>1370</td>
<td>960</td>
<td>530</td>
</tr>
<tr>
<td>1.22 (48)</td>
<td>1020</td>
<td>680</td>
<td>400</td>
</tr>
</tbody>
</table>

The lamps are the 40-watt, 48" $T-12$, 2500K white fluorescent lamp (3800 lumens), and the T-8, 96" 4500K white lamp operating at 200 ma (3850 lumens) and 100 ma (1800 lumens). Reflectances are $\rho_w = 0.30$, $\rho_i = 0.50$, $\rho_e = 0.80$. Also, $\tau = 0.40$, $k_\rho = 0.87$, $k_\nu = 0.76$ for $k_r \to 0$, $k_\nu = 0.33$ for $k_r = 0.70$.

---

**TABLE IIb — Average Illumination (Pharosage) as a Function of Lamp Spacing**

<table>
<thead>
<tr>
<th>Normal Spacing (m)</th>
<th>Average Illumination (Pharosage) (lumen ft$^{-1}$) in Service</th>
<th>$k_r \to 0$</th>
<th>$k_r = 0.70$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.30 (12)</td>
<td>380</td>
<td>250</td>
<td>150</td>
</tr>
<tr>
<td>0.40 (18)</td>
<td>250</td>
<td>170</td>
<td>100</td>
</tr>
<tr>
<td>0.61 (24)</td>
<td>190</td>
<td>120</td>
<td>74</td>
</tr>
<tr>
<td>0.76 (30)</td>
<td>150</td>
<td>100</td>
<td>60</td>
</tr>
<tr>
<td>0.92 (36)</td>
<td>130</td>
<td>84</td>
<td>50</td>
</tr>
<tr>
<td>1.22 (48)</td>
<td>95</td>
<td>63</td>
<td>37</td>
</tr>
</tbody>
</table>

---

Lighting in accordance with the foregoing specifications has been installed in one room, and other installations are in the process of construction. Tests indicate that the theoretical analysis is sound. There remain many questions of best ar-
rangements for practicability and economy, and it is hoped that the present paper will stimulate further work in the development of ceiling lighting.

**APPENDIX A**

**INTERFLECTIONS IN AN INFINITE ROOM**

Consider an infinite room (Fig. 4) lighted by means of a luminous ceiling. It is proposed to investigate the interreflections in the complete assembly. All surfaces are assumed to be perfectly diffusing, and \((F_L/S)\) is the total lamp lumens (pharos) per unit area of ceiling.

(I) First we formulate the interreflections in the room itself, with arbitrary incident illumination (pharosage) \(D_0\) on the top of the translucent layer and no interreflections in the luminaire.

Fig. 12 gives a schematic diagram of the multiple reflections within the room. Adding all the components that go upward through the translucent plate, we obtain the brightness (helios) of the plate as seen from above:

\[
H_a = D_0 \left[ \frac{\rho_2 + \rho_3 \tau^2}{1 - \rho_2 \rho_3} \right].
\]

But \(H_a/D_0 = \rho^* = \text{apparent reflectance of plate, as seen from above.}\) Thus when light is incident on the upper side of the translucent plate (or "diaphosphor"), it is not reflected according to the usual reflectance \(\rho_2\) of this plate but according to the enhanced reflectance,

\[
\rho^* = \frac{\rho_2 + \rho_3 \tau^2}{1 - \rho_2 \rho_3}. \tag{9}
\]

The apparent reflectance \(\rho^*\) is higher than \(\rho_2\) because of the light reflected within the room.

Similarly, the brightness (helios) of the translucent plate, as seen from below, is obtained by adding all the contributions (Fig. 12), or

\[
H_b = \tau D_0 \left[ 1 + \rho_2 \rho_3 + (\rho_2 \rho_3)^2 + \cdots \right] = \tau D_0 \left[ \frac{\rho_2 + \rho_3 \tau^2}{1 - \rho_2 \rho_3} \right].
\]

But \(H_b/D_0 = \tau^* = \text{apparent transmittance of ceiling, with no interreflections within the luminaire.}\) Thus,

\[
\tau^* = \frac{\tau}{1 - \rho_2 \rho_3}. \tag{10}
\]

The apparent transmittance of the diaphosphor is higher than the measured transmittance \(\tau\) because of multiple reflections within the room.

(II) The second step is to employ equations (9) and (10) in calculating the interreflections in the luminaire (Fig. 13). Irrespective of where the fluorescent lamps are placed in the luminaire, exactly half of their luminous output is incident directly on the ceiling and half on the translucent plate. The total incident illumination (pharosage) without multiple reflections, is evidently

\[
D_0 = \frac{F_L}{2S} (1 + \rho_2).
\]

Adding the components reflected upward (Fig. 13), we obtain

\[
H_a = D_0 \tau^* \left[ 1 + (\rho^* \rho_3) + (\rho^* \rho_3)^2 + \cdots \right] = \frac{D_0 \tau^*}{1 - \rho^* \rho_3} = \frac{F_L (1 + \rho_2) \rho^*}{2S (1 - \rho^* \rho_3)}. \tag{11}
\]

And the transmitted components give

\[
H_b = D_0 \tau^* \left[ 1 + (\rho^* \rho_3) + (\rho^* \rho_3)^2 + \cdots \right] = \frac{D_0 \tau^*}{1 - \rho^* \rho_3} = \frac{F_L (1 + \rho_2) \rho^*}{2S (1 - \rho^* \rho_3)}. \tag{12}
\]

Also,

\[
H_b = \frac{\rho_2 F_L}{2S} + \frac{\rho^* \rho_3 D_0 [1 + (\rho^* \rho_3) + (\rho^* \rho_3)^2]}{2S (1 - \rho^* \rho_3)} = \frac{\rho_2 F_L}{2S} \left[ 1 + \frac{\rho^*(1 + \rho_2)}{1 - \rho^* \rho_3} \right]. \tag{13}
\]

(III) We now obtain equations for coefficient of utilization, logance, and apparent reflectance of the complete luminous ceiling. Since the room is infinite in extent, the average illumination (pharosage) on the principal surface is

\[
D_{AV} = H_a.
\]

or from equation (12),

\[
D_{AV} = \frac{(1 + \rho_2) \tau^*}{2S (1 - \rho^* \rho_3)} \frac{F_L}{S} = \frac{\tau (1 + \rho_2)}{2 (1 - \rho_2 \rho_3)} \frac{F_L}{S} \tag{14}
\]

But in any room,

\[
D_{AV} = \frac{F_L}{S}.
\]
Thus the coefficient of utilization \( k_u \) for an infinite room with Type IIIa lighting is

\[
k_u = \frac{\tau (1 + \rho_a)}{2(1 - \rho_2 \rho_a) \left( 1 - \rho_a \right) \left( 1 - \rho_a \rho_2 \rho_a \right) \left( 1 - \rho_2 \rho_a \rho_2 \rho_a \right)}
\]

\[
= \frac{\tau (1 + \rho_a)}{2(1 - \rho_2 \rho_a) \left( 1 - \rho_2 \rho_2 \rho_a \right) \left( 1 - \rho_2 \rho_a \rho_2 \rho_a \right)}
\]

Comparison of equations (3) and (6) shows that the efficiency (logance) of the luminaire is

\[
\theta = \frac{\tau (1 + \rho_a)}{2(1 - \rho_2 \rho_a) \left( 1 - \rho_2 \rho_2 \rho_a \right) \left( 1 - \rho_2 \rho_a \rho_2 \rho_a \right)}
\]

(7)

and the apparent reflectance of the translucent ceiling, as seen from below, is

\[
\rho_2^a = \frac{\rho_2 + \rho_2 \tau^2}{1 - \rho_2 \rho_a}
\]

(8)

A comparison of equations (8) and (9) indicates that the apparent reflectance of the translucent ceiling is different when viewed from above and from below. The two expressions differ only in the substitution of \( \rho_a \) instead of \( \rho_2 \). It is easy to show that

\[
(1 - \rho_2 \rho_a) \left( 1 - \rho_2 \rho_2 \rho_a \right) = (1 - \rho_2 \rho_2 \rho_a) \left( 1 - \rho_2 \rho_a \rho_2 \rho_a \right)
\]

(16)

**APPENDIX B**

**BRIGHTNESS (HELIOS) UNIFORMITY FOR INFINITE ROOM**

The pharosage from a single infinite row of uniform fluorescent lamps is

\[
D = \frac{F_L \cos^2 \theta}{2 \pi S}
\]

where the quantities are indicated in Fig. 2. But

\[
\cos^2 \theta = \frac{1}{1 + (s/l)^2}
\]

Thus for \( \theta = 0 \), the incident pharosage produced by an infinite number of lamps (no interferences), with uniform spacing \( s \) between rows, is

\[
D_{\text{max}} = \frac{F_L (s/l)}{2 \pi S} \left[ 1 + 2 \sum_{n=1}^{\infty} \frac{1}{1 + (n s/2l)^2} \right]
\]

(17)

Midway between rows of lamps,

\[
D_{\text{min}} = \frac{F_L (s/l)}{2 \pi S} \left[ 1 + \sum_{n=1}^{\infty} \frac{1}{1 + (n s/2l)^2} \right]
\]

(18)

Equations (17) and (18) give the illumination (pharosage) produced directly by the lamps. To these values must be added the illumination (pharosage) from the upper surface of the luminaire. This added illumination (pharosage) is equal to \( H_0 \) and is assumed to be uniform over the entire diaphragm. Then, from equations (12), (17), and (18),

\[
e = \frac{1 - \sum \left( \frac{1}{1 + (n s/2l)^2} \right)}{1 + \sum \left( \frac{1}{1 + (n s/2l)^2} \right) + \frac{A}{(s/l)}}
\]

(19)

where

\[
A = \pi \rho_5 \left[ 1 + \frac{\tau (1 + \rho_a)}{1 - \rho_2 \rho_a} \right]
\]

These equations were used in computing the curves of Fig. 8.

**APPENDIX C**

**BRIGHTNESS (HELIOS) VARIATION, SMALL NUMBER OF ROWS**

Consider now the incident illumination (pharosage) on the top of the translucent plates, produced directly by a small number of rows of lamps, neglecting interferences. The incident illumination (pharosage) produced by one row of lamps is

\[
D = \frac{F_L (s/l)}{2 \pi S} \left[ 1 + \frac{1}{1 + (x/l)^2} \right]
\]

Directly beneath the lamp, \( x = 0 \) and the pharosage is

\[
D_1 = \frac{F_L (s/l)}{2 \pi S}
\]

At any other point,

\[
\frac{D}{D_1} = \frac{1}{1 + (x/l)^2}
\]

(20)

Equation (20) was used in plotting the curve for a single row of lamps. Superposition of these curves gave Figs. 9 and 10.

**References**